Abstract—The restoration process of a power system after a blackout consists of three phases, namely starting up the generators, re-energizing the network, and picking the loads. The generator start-up sequence is pivotal for the total restoration time and the following restoration steps. In this paper, a novel algorithm for optimizing a generator start-up sequence is proposed. Based on the VIKOR method, which is a multi-objective approach, the proposed algorithm is not only able to improve the start-up time and reliability of a generator start-up sequence, but can also handle auxiliary optimization criteria with priorities that change during the start-up. Such criteria are, for instance, the importance of the power increasing rate, reliability and surrounding topologies of generators. The efficiency of the proposed method is evaluated in two tests. In the first test exhaustive search is used to compute optimal solutions. In the second and larger test NSGA-II is utilized to approximate the Pareto frontier. For both tests the proposed algorithm approximates the reference results while being computationally very efficient. This makes it suitable for online decision making where a new start-up sequence has to be computed in a few seconds or minutes.

Index Terms—Network Restoration, Generator Start-Up Sequence, VIKOR Algorithm, Analytic Hierarchy Process (AHP), Performance Index

I. INTRODUCTION

With the development of the power system, the network becomes more and more flexible and robust. However, the risk of a total power system blackout is still present and with the increasing share of renewable energy sources even on the rise. A power system blackout can cause dramatic consequences. Recent power system blackouts (for instance, the Northeast America blackout in 2003 [1], the Japan power system collapse caused by an earthquake in 2011 [2] and the Northern, Eastern and Northeast India power system blackout in July 2012 [3]) have demonstrated that an efficient power system restoration plan is of utmost importance.

Generally, the restoration process consists of three phases: the start-up of generators, the re-energizing of the network, and the restoration of load. In related work, general guidelines for the power system restoration are given by [4], [5]. An automatic approach is introduced in [6], [7], [8], [9] by implementing a rule-based expert system to establish a dynamic restoration plan considering variations in the restoration process.

In [10] the generator start-up sequence problem is formalized as a mixed integer linear programming (MILP) problem. However, this paper considers only the time constraints. Important performance indexes such as the power increasing rate, capacity, reliability and node importance degree of the non-black start units have not been considered. In [11] and [12] the analytic hierarchy process (AHP) has been introduced using vague set theory for the optimization the start sequence of non-black start units. The approach considers only the static assessment of the performance indexes. However, different generator start-up stages require for different performance index configurations. For instance, in the initial phase of the restoration process the network is vulnerable. Thus, the non-black start units with high reliability index should be restarted first. In the following stage, the network is robust enough to withstand a singular generator failure. Therefore, non-black start units with high power increasing rates, low cranking powers, and low restart waiting times should be preferred to shorten the overall start-up process. In the last stage, the locations (node importance degrees) of non-black start units became the most important factors.

In this work we extend the analytic hierarchy process having an expert database covering three different network restart stages. The database includes only major decision variables but can be extended easily to the concrete requirements of a network provider. The goal is to show that the methodology of a priory collecting how experts would decide in some situations and then to use this data to automatically and instantaneously derive a decision which power plant to boot next, combined with a very fast multi-objective algorithm, can create a tool that can consider also the current network status and can offer optimized choices to the network engineer how to proceed next during the restoration.

This paper proposes a novel method based on VIKOR multi criteria decision-making algorithm [13] to optimize the generator start-up sequence considering different performance indexes of non-black start units. Furthermore, the temporal priority variations of performance indexes in different start-up stages are modeled by time weighting vectors.

II. FORMALIZATION OF THE START-UP SEQUENCE

The identification of a good generator start-up sequence has to consider many aspects, such as the minimal and maximal restart times of non-black start units, the power increasing rates, the reliability and capacity of different units. In this work, a start-up sequence is modeled by two goal functions, three constraint functions, and seven performance indexes.
A. Objective Functions

The one of the most important tasks of the restoration process is to maximize the reliability index of the generator start-up sequence minimizing the possibility that the power system collapses again. The second task is to achieve the minimal generator start-up time. Therefore, the task of this paper is to arrange the generator start-up sequence such that the reliability index is maximized: \( f_1 = \max R \), with \( R \) as the reliability index, and the restoration time is minimized: \( f_2 = \min T \), with \( T \) as the execution time for a particular start-up sequence. Operation time of switches and switch insulators is short compared to the time for a generator start-up and is neglected in this paper.

B. Constraints

Following constraints have to be respected during the restoration process:

1) All black start units have to have enough accumulated power \( P^B \) to restart any non-black start unit with the cranking power of \( P^{CNB} \). \( P^B \geq P^{CNB} \).
2) A non-black start unit can receive its cranking power \( P^{CNB} \) for at least \( T_{min} \) time to be able to start.
3) Since cranking power consists mainly of induction motors, it is important to ensure that node voltages and network frequency lie in acceptable ranges when black start units send cranking power to non-black start units.

C. Performance Indexes

In order to evaluate the performance of non-black start units, the definitions of following performance indexes are introduced:

1) Reliability index \( R_i \) defines the annual reliability index of \( i \)-th non-black start unit. Non-black start units with higher reliability indexes should have higher priority for being selected first for restart to ensure the power system security.
2) According to [14], the importance degree \( \alpha_i \) of a generator \( i \) corresponds to the surrounding network topology and is defined as \( \alpha_i = \frac{1}{n_i l_i} \), with
   \[
   n_i = \sum_{j \in v} d_{min,ij} / n_i (n_i - 1)/2
   \]
   where \( n_i \) is the total number of nodes in the new network after the contraction of node \( i \), \( l_i \) is the average of the shortest distances in the new network after the contraction of node \( i \), and \( d_{min} \) is the shortest distance between node \( i \) and \( j \) counted in the number of branches after the contraction of node \( i \).
3) Restart of large non-black start units can accelerate the total restoration process and pick up more important load in the initial restoration phase. The larger the capacity \( C_i \) of a non-black start \( i \) unit is, the higher its priority for being selected for restart.
4) The power increasing rate \( I_i \) of a non-black start unit \( i \) is reciprocal to the restoration time. In order to achieve the minimal restoration time, non-black start units with higher power increasing rates should be restarted first.
5) The maximal restart time \( T_{max} \) defines the time period a non-black start unit \( i \) has to receive its cranking power \( P^{CNB} \) uninterruptedly for a reboot. Otherwise the restarting process of \( i \) is delayed by several hours.
6) After receiving the cranking power within \( T_{max} \), the non-black start unit \( i \) need some preparation time \( SP_{i} \) to restart its ancillary devices and start injecting energy into the network.
7) A non-black start unit with low cranking power \( C_i \) can be restarted quickly and should be considered first for reboot.

III. PROPOSED METHOD

The goal of the proposed method is to find a generator start-up sequence with a short restoration time and good restoration reliability while considering three temporal restoration phases with different performance index priority combinations. The overall structure of the algorithm is presented in Fig. 1.

First, the algorithm computes seven performance weighting factors using the AHP method. AHP is a popular approach to transform expert knowledge given as a list of importance rankings for all decision criteria pairs into a non-contradicting importance ranking matrix and an overall decision weighting factor. For the three temporal network restoration phases, Tab. I presents the AHP harmonized results. A value greater or lower than one indicates that the performance index in the according row is more or less important than the performance index in the according column, respectively. The last three columns contain the normalized decision vectors for all three restoration phases.
In the next step the performance indexes are transformed into fuzzy membership functions. The VIKOR algorithm that relies on a fuzzy parameter set requires this. Tab. III shows the middle values of the triangular fuzzy membership functions. For simplicity, the lower and upper fuzzy membership function values are computed by subtracting and adding 0.05 to the middle values. However, any valid fuzzy membership function parameterization can be used.

Finally, the adopted VIKOR method is applied to the seven performance weighting factors, the fuzzy membership functions, and the two goal functions. In the adopted version of the VIKOR algorithm proposed in this paper the trade-off solutions are computed independently for each start-up stage. Then the goal functions are evaluated and averaged. The final step of the VIKOR algorithm, the ranking of solutions according to the fitness comparison matrix, remains unmodified, and the algorithm returns the best generator start-up sequence according to the computed ranking.

### IV. Case Study

For a sound evaluation of the proposed method the paper defines two test cases A and B. The first test case defines a small network restoration instance. With this, exhaustive search can be used to parse all valid start-up sequences and identify Pareto solutions (global optima). The results can then be compared to the results of the proposed approach and the absolute quality distances can be computed. For case B the paper extends the network from case A such that exhaustive search becomes infeasible due to the larger search space. Now a state-of-the-art Pareto-based multi-objective evolutionary algorithm NSGA-II is used to approximate the Pareto frontier. Again, the results are compared to the results of the proposed approach and quality distances can be computed.

Both test cases rely on the expert knowledge balancing the importance of the performance indexes for the three temporal network restoration phases. Tab. I summarizes the harmonized pairwise importance rankings for all seven performance indexes and phases. For instance, the importance of the reliability of generators \( R_t \) is pivotal in the first phase and descends in the second and third phases. This can be observed in the first line of Tab. I. When comparing \( R_t \) to the node importance degree \( \alpha_i \), the table reveals the tuple \([2.0 \quad 1.2 \quad 0.5]\) for the first, second and third network restoration phases. That means that in the first phase \( R_t \) is as twice as important than \( \alpha_i \). The importance of \( R_t \) declines until \( R_t \) is only half as important as \( \alpha_i \) in the third phase. This impact behavior of \( R_t \) can be observed also for the pairwise comparisons with other performance indexes in the first line of Tab. I. Other lines of Tab. I present the impact behaviors for the remaining performance indexes.

In the last three columns of Tab. I the normalized decision weighting vectors for all performance indexes and the three temporal network restoration phases are presented. Computed by AHP, these values transform the pairwise importance rankings of the performance indexes to a normalized decision vector establishing a global balance between the performance indexes.

#### Case A

Test case A uses the topology of the new England 39-bus test network [15]. The black and non-black start units have been parameterized according to typical configurations of hydroelectric power plants in a 220kV and steam turbines
in a 220kV/110kV network, respectively. The parameters are presented in Tab. II. Altogether, there are ten generators in the network. Generators 1 to 3 are black start while 4 to 10 are non-black start units. According to the proposed method, for all generators and performance indexes the membership functions are computed and presented in Tab. III. The lower and upper fuzzy values of the membership function can be obtained by subtracting and adding 0.05 to the middle values in Tab. III.

First, the proposed algorithm is evaluated on the complete network restoration using either the decision vectors for the first, second or the third phases. This helps getting a rough insight into the variety of potential solutions. The three resulting start-up sequences and their restoration times and reliabilities are presented in the first three lines of Tab. IV. As expected, with the decreasing impact of the reliability index $R_i$ from 0.22 over 0.13 to 0.1 in the first, second and third network restoration phases, as presented in the first line of Tab. I, the reliability of the computed generator start-up sequences declines from 79.36% over 78.57% to 77.5%. The variation is small due to the size of the used network topology. This is also the case for the network restoration times in the last column of Tab. IV, where all algorithm runs can restart the network in 230 minutes.

As there is no strict global temporal subdivision between the three network restoration stages, namely the transition from booting reliable non-black start units to booting non-black start units with high power increasing rates and the succeeding transition to booting closely located non-black start units, this paper evaluates multiple temporal subdivision schemes. While fixing the impact of the third restoration phase to 0.1, the impact of the first restoration phase is increased from 0.1 over 0.3 and 0.5 to 0.8. At the same time, the impact of the second restoration phase is decreased from 0.8 over 0.6 and 0.4 to 0.1. The results can be seen in the fourth to seventh lines of Tab. IV. For all time weighting vectors the algorithm was able to restore the network in 230 minutes. Except for the [0.1 0.8 0.1] time weighing vector, the algorithm achieves for all other combinations the restoration reliability of 78.82%. Finally, a combination of equally weighted restoration stages is evaluated. The results in line eight shows that this combination also achieves reliability of 78.82% and a restoration time of 230 minutes.

The small size of the investigated network allows enumerating all valid start-up sequences in a reasonable time. Using this exhaustive search method, 5040 possible start-up sequence combinations for 7 non-black start units have been evaluated. After approximately 6 minutes of computing time 400 start-up sequences with a restoration time of 210 minutes have identified. Thereby, reliability of 78.0% was not exceeded. The most reliable network restoration with 80.18% was achieved by the start-up sequence [5 7 10 9 4 8 6] in 230 minutes. This is 1.02% better than the results achieved by the proposed algorithm. The results of the proposed algorithm and the exhaustive search are presented in Fig. 2.

**Case B**

For the second test, the test network has been extended by five additional non-black start units. The parameters of the new generators and their fuzzy membership function values are illustrated in lower parts of Tab. II and III.

The evaluation methodology for case B is identical to case A except that exhaustive search cannot be used anymore due to the excessive computation complexity. Instead, NSGA-II is employed for Pareto frontier approximation. NSGA-II has been configured to work on a population of 100 individuals with a mutation and recombination rates of 0.1 and 0.8 and to stop after 100 generations.

The results are presented in the lower part of Tab. IV. Due to the larger network the results are more diverse than in case A. The results show clearly the correlation of different weighting factors preferring either the reliability $R_i$ (phase 1) or power increasing rates $I_i$ (phase 2) on the reliabilities and network restoration times of the resulting start-up sequences. The non-dominated results and the results of NSGA-II are also presented in Fig. 2. One can observe that both algorithms are able to approximate the Pareto frontier. However, NSGA-II is evolving a dominant (better) set of solution than the proposed method. But there is a crucial difference between the algorithms. NSGA-II requires more than 3 minutes to solve the generator scheduling challenge while the proposed method lies well below a second to find a single restoration plan. With this, executing the proposed method with different time weighting vectors one can approximate the Pareto frontier faster.

| TABLE III: Middle values for the triangular fuzzy membership functions of non-black start units for case A (4—10) and B (4—15) |
|-----------------|---------|---------|---------|---------|---------|---------|---------|
|                | $R_i^m$ | $R_i^m$ | $C_i^m$ | $I_i^m$ | $T_{max}$ | $S_{max}$ | $C_i^m$ |
| 4               | 0.72    | 0.63    | 0.77    | 0.64    | 0.50    | 0.92    | 0.84    |
| 5               | 0.86    | 0.52    | 0.82    | 0.92    | 0.86    | 0.64    | 0.73    |
| 6               | 0.62    | 0.53    | 0.86    | 0.59    | 0.50    | 0.50    | 0.67    |
| 7               | 0.65    | 0.53    | 0.95    | 0.71    | 0.50    | 0.57    | 0.50    |
| 8               | 0.66    | 0.61    | 0.82    | 0.79    | 0.50    | 0.94    | 0.73    |
| 9               | 0.77    | 0.51    | 0.82    | 0.70    | 0.77    | 0.92    | 0.73    |
| 10              | 0.83    | 0.68    | 0.86    | 0.74    | 0.94    | 0.85    | 0.67    |
| 11              | 0.76    | 0.48    | 0.86    | 0.70    | 0.50    | 0.82    | 0.81    |
| 12              | 0.93    | 0.48    | 0.99    | 0.81    | 0.50    | 0.57    | 0.41    |
| 13              | 0.96    | 0.48    | 0.91    | 0.86    | 0.50    | 0.50    | 0.57    |
| 14              | 0.69    | 0.48    | 0.52    | 0.58    | 0.50    | 0.82    | 0.87    |
| 15              | 0.65    | 0.48    | 0.50    | 0.53    | 0.50    | 0.86    | 0.91    |

**Calculation time analysis**

Tab. V presents the calculation times for the NSGA-II and the proposed method for different power grid sizes. Both algorithms are implemented in Matlab and executed by a computer with a Core i7 2.66 GHz processor. The execution times can be significantly improved if implementing the algorithms in, e.g., the language C. However, even using the not optimized Matlab implementation the performance of our method is sufficient for online decision making, even if applied on larger networks.

**V. CONCLUSION AND OUTLOOK**

This paper introduces a new way of optimizing the generator start-up sequence considering different and changing requirements during a network restoration. To this, an algorithm based
on the analytic hierarchy process and an adopted VIKOR method is implemented. The algorithm is able to approximate the Pareto frontier. This has been shown in two benchmarks. In the first test case exhaustive search has been used to establish the Pareto frontier while in the second and larger test case the Pareto frontier has been approximated by NSGA-II. The pivotal property of the proposed algorithm is its computational efficiency, which allows using it for online decision making during a network restart, thus quickly adapting to rapid changes. For larger and more realistic networks, exhaustive search and NSGA-II may require even more computation time.

This research is part of a project aiming at the development of a new power system black start algorithm. After having established a method for rebooting the generators the next step is to develop an optimized load restoration algorithm considering load types, excitations and control systems of generators.

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